

Localization for Wireless Sensor Networks

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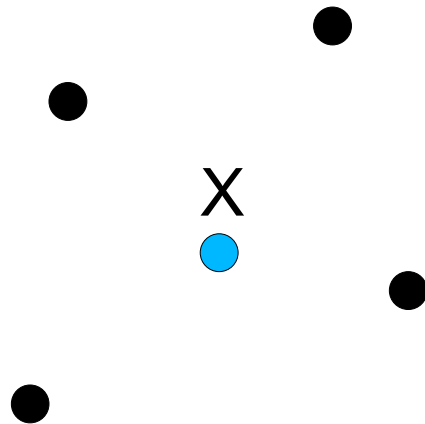
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Localization Techniques

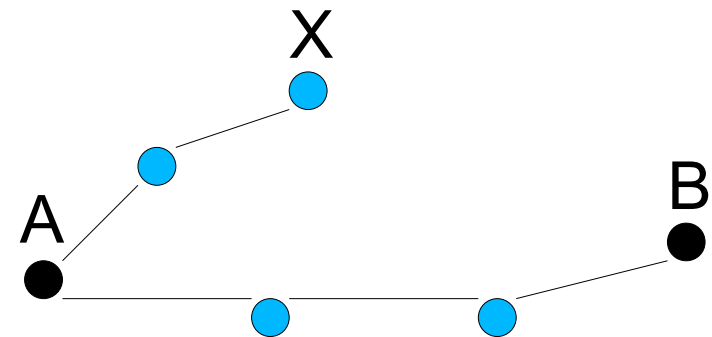
- Range-free Methods
 - Depends on a good geographical position of the anchor nodes
- Anchor Nodes
 - Nodes that are aware of their positions
- Range Measurements
 - Method to estimate the distance or angle between a pair of nodes

Range-free Methods

■ Centroid



■ DV-HOP



$$d_{AB} = 60m \Rightarrow d_{AX} = 40m$$

$$(x,y) = \left(\frac{x_1 + \dots + x_n}{n}, \frac{y_1 + \dots + y_n}{n} \right)$$

Range Measurements

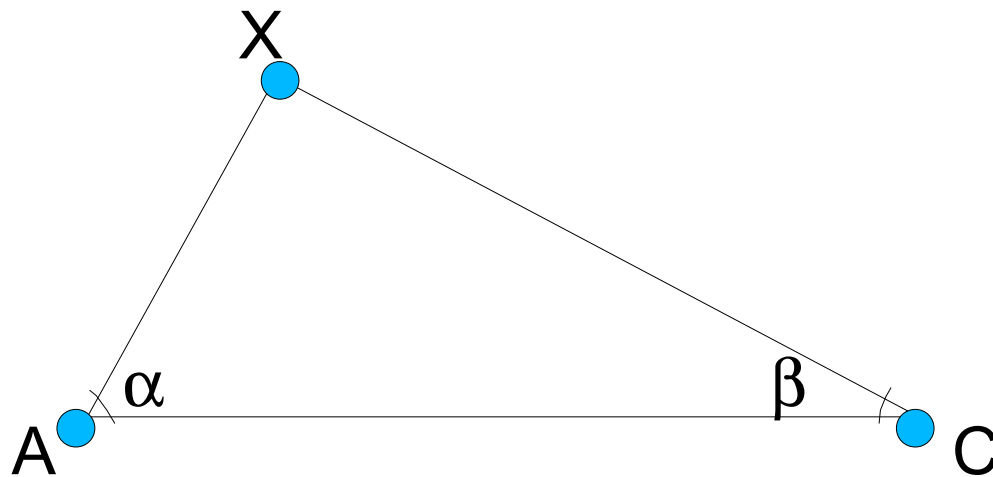
- ToA (Time of Arrival)
 - Requires accurate clocks
- TDoA (Time Difference of Arrival)
 - Requires additional hardware (ultra-sound device)
- AoA (Angle of Arrival)
 - Requires additional hardware (for angle estimation)
- RSSI (Received Signal Strength Indication)
 - No additional hardware
 - Imprecise

Position Calculation

- Triangulation
- Trilateration
- Multilateration

Triangulation

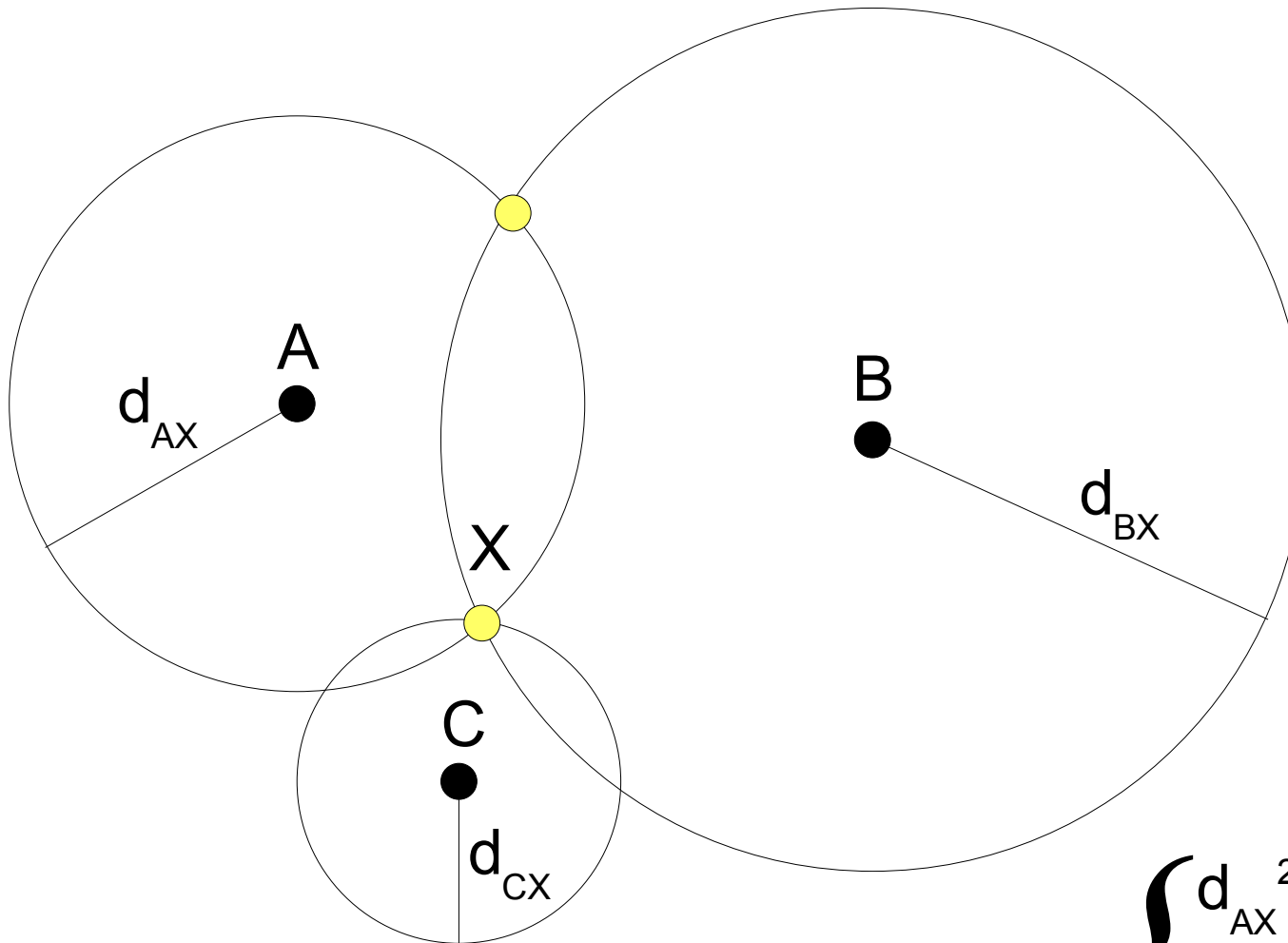
- Angle information is provided.
 - Coordinates can be defined through trigonometry rules (law of sines, law of cosines, trigonometric functions).



$$m = \operatorname{tg} \alpha \quad n = \operatorname{tg} \beta$$

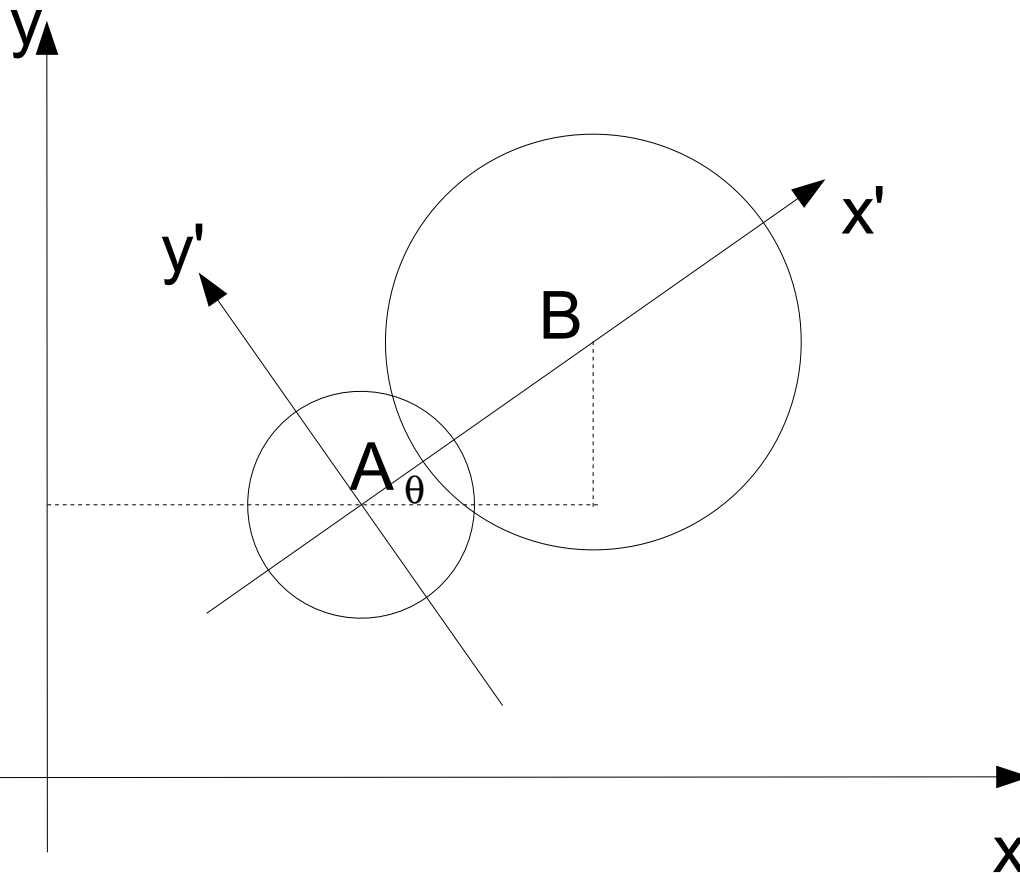
$$\begin{cases} y = m (x - x_A) + y_A \\ y = n (x - x_B) + y_B \end{cases}$$

Trilateration



$$\begin{cases} d_{AX}^2 = (x-x_A)^2 + (y-y_A)^2 \\ d_{BX}^2 = (x-x_B)^2 + (y-y_B)^2 \\ d_{CX}^2 = (x-x_C)^2 + (y-y_C)^2 \end{cases}$$

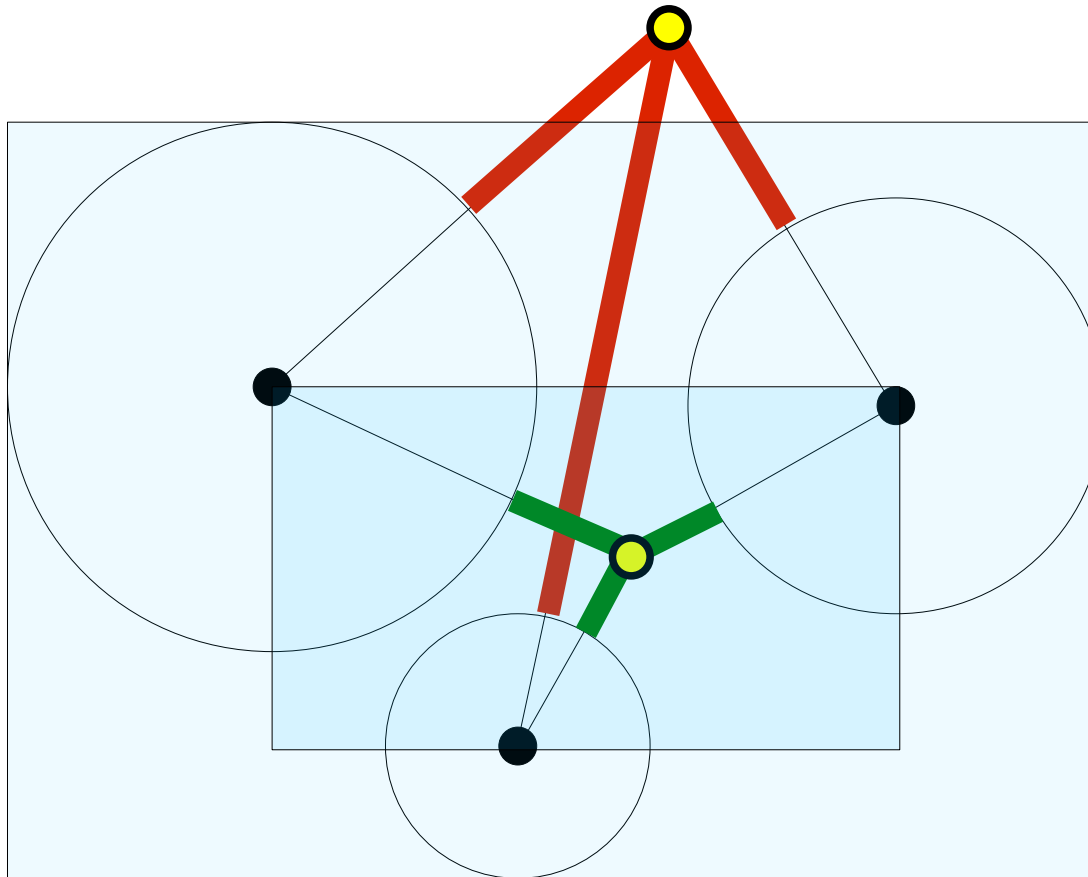
Translation and Rotation



$$x' = x \cos \theta - y \sin \theta$$
$$y' = x \sin \theta + y \cos \theta$$

$$\sin \theta = (y_B - y_A) / d_{AB}$$
$$\cos \theta = (x_B - x_A) / d_{AB}$$

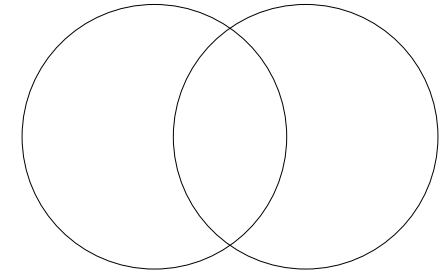
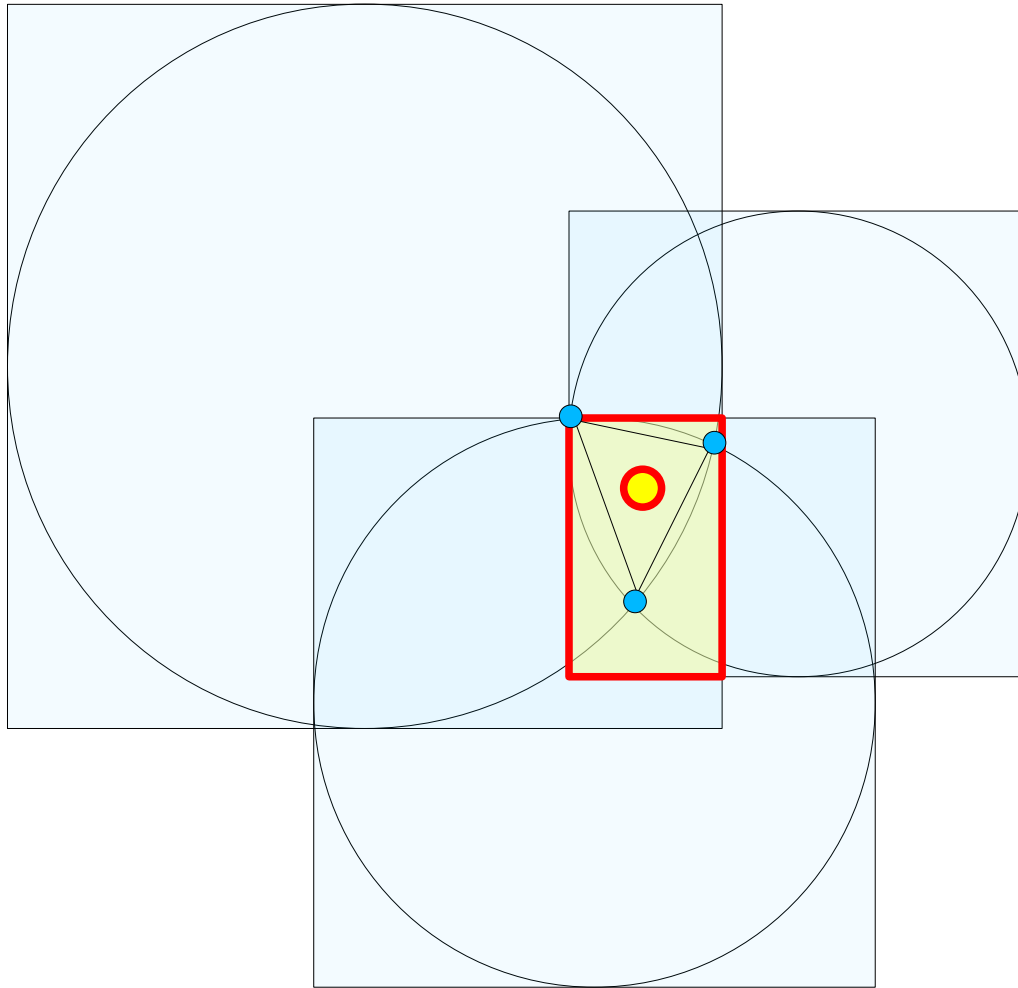
Approximation



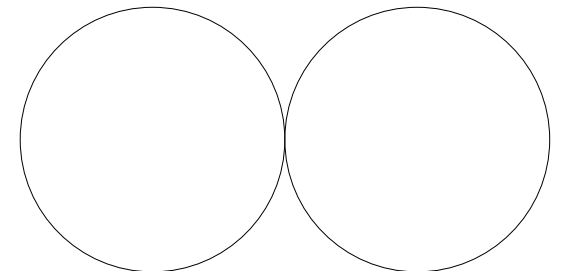
$$\text{Residue} = \sum_{i=1}^n |d_{ix} - d_x|$$

Where i iterates through anchors (and n is the total number)

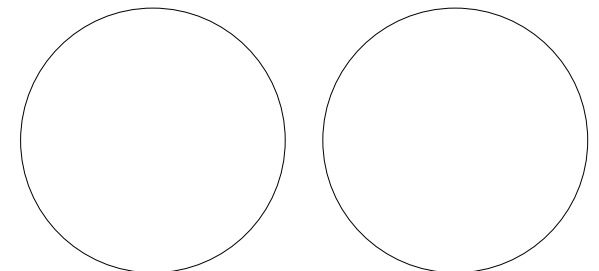
Min-Max



$$d_{AB} < r_A + r_B$$



$$d_{AB} == r_A + r_B$$



$$d_{AB} > r_A + r_B$$

Multilateration

$$\begin{cases} (x_1 - x)^2 + (y_1 - y)^2 = d_1^2 \\ \vdots \\ (x_n - x)^2 + (y_n - y)^2 = d_n^2 \end{cases}$$

The system can be linearized by subtracting the last equation from the first $n-1$ equations. Reordering the terms gives a proper system of linear equations in the form of $Ax = b$, where:

$$A = \begin{bmatrix} 2(x_1 - x_n) & 2(y_1 - y_n) \\ \vdots & \vdots \\ 2(x_{n-1} - x_n) & 2(y_{n-1} - y_n) \end{bmatrix}$$

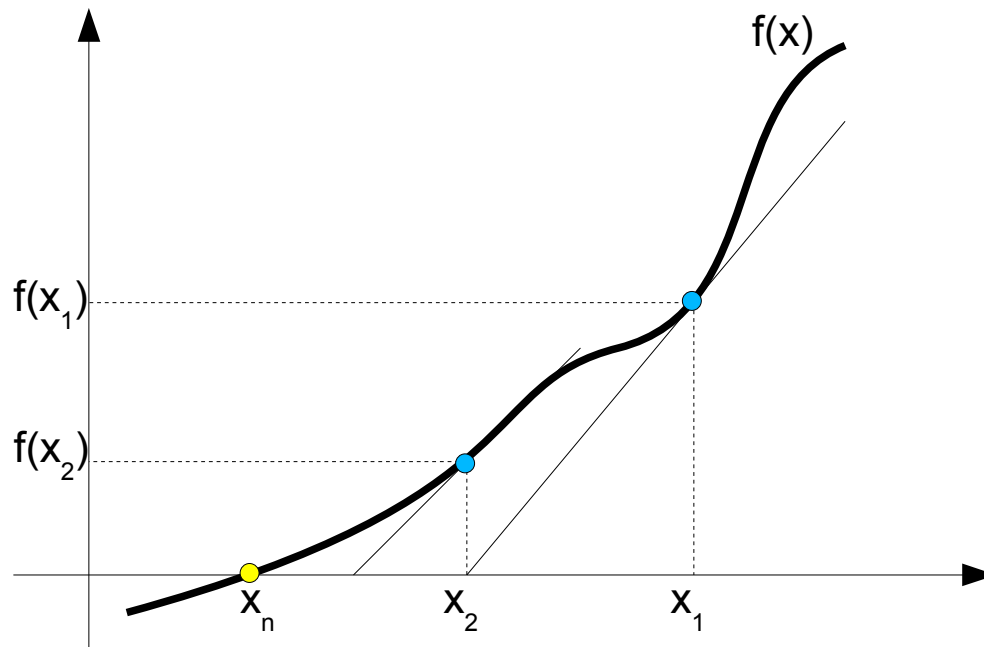
Using a standard least-squares approach:

$$\hat{x} = (A^T A)^{-1} A^T b$$

$$b = \begin{bmatrix} x_1^2 - x_n^2 + y_1^2 - y_n^2 + d_n^2 - d_1^2 \\ \vdots \\ x_{n-1}^2 - x_n^2 + y_{n-1}^2 - y_n^2 + d_n^2 - d_{n-1}^2 \end{bmatrix}$$

Square-root Approximation

- Newton-Raphson method to find roots of functions

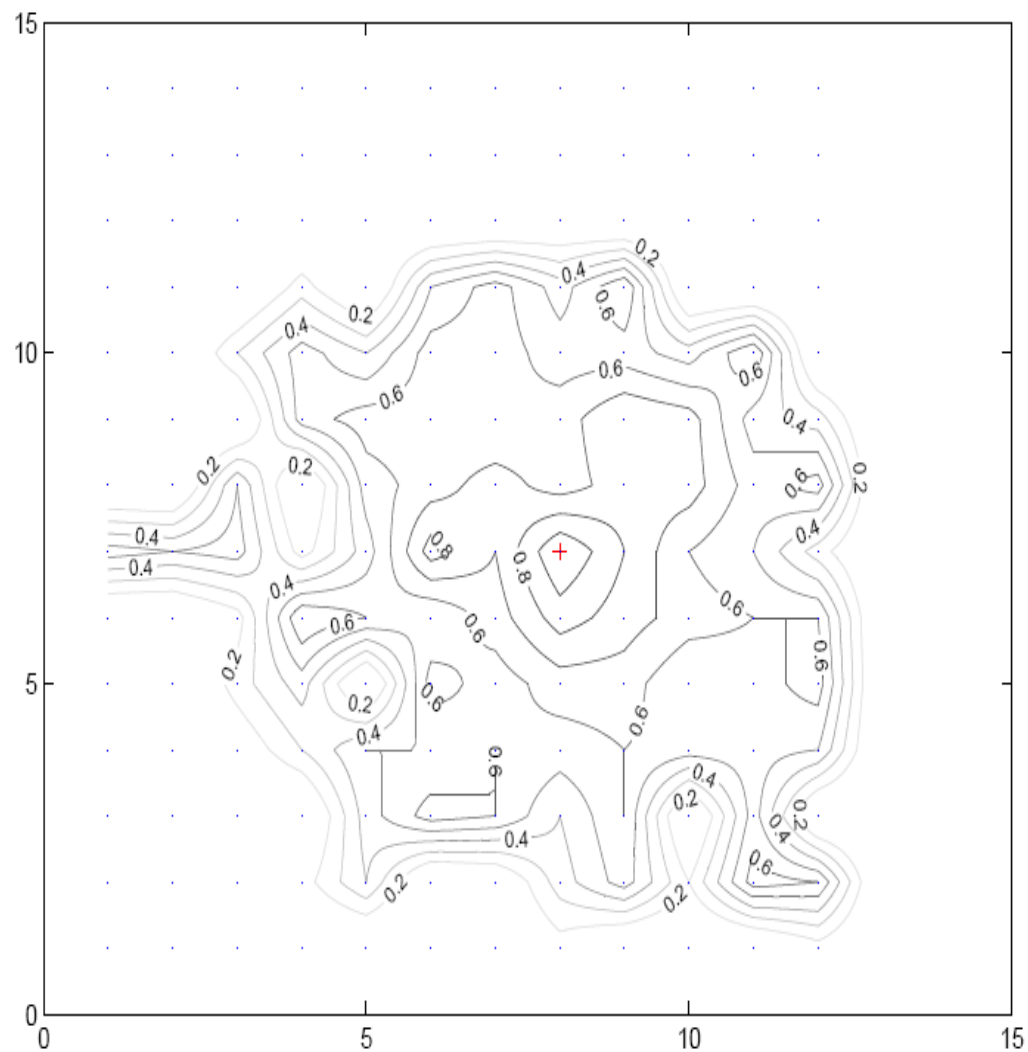
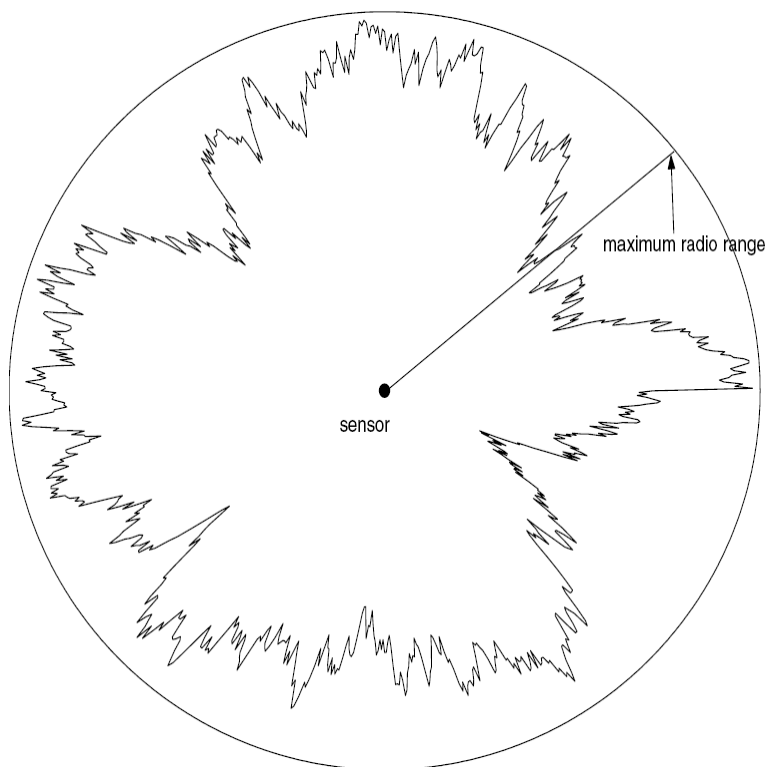


$$f'(x_n) = f(x_n) / (x_n - x_{n+1})$$

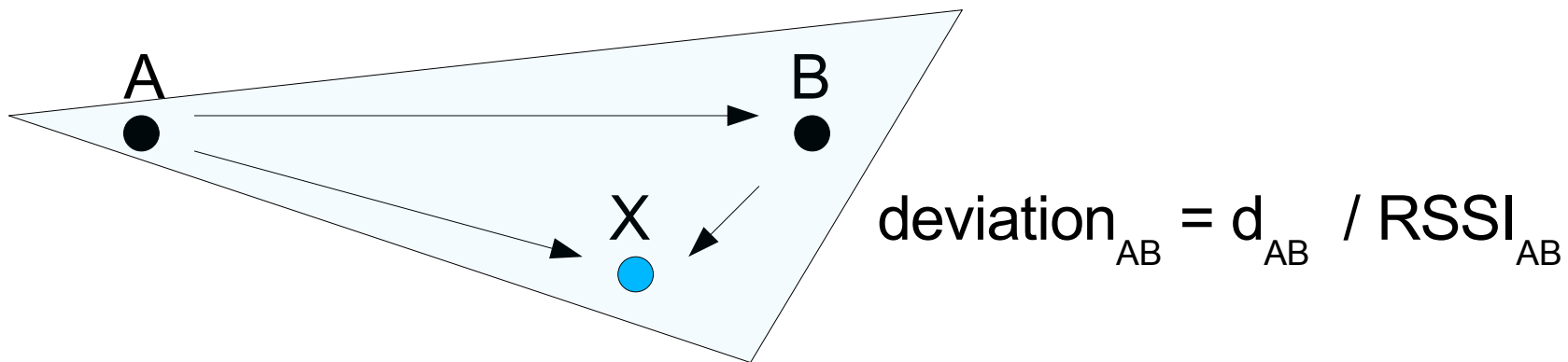
$$x_{n+1} = x_n - f(x_n) / f'(x_n)$$

- Which function gives the square-root of a number?

RSSI Fluctuations



HECOPS Calibration



$$d_{AX} = \text{deviation}_{AB} \cdot \text{RSSI}_{AX}$$