Computer Organization and Programming

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Outline

- Computer Organization and Programming
  - Data Representation
  - Boolean Algebra
  - Basic Computing System Components
  - Instruction Set Architecture
  - Program translation into machine language (lab)
Numbering Systems

- **Decimal System (humans)**
  - **Base 10**
    
    \[
    123 = 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0
    \]
    
    \[
    123.456 = 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 6 \times 10^{-3}
    \]

- **Binary System (computers)**
  - **Base 2**
  - Represented in digital systems by two voltage levels
    
    \[
    11001010 = 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 128 + 64 + 8 + 2 = 202
    \]
Numbering Systems

- Hexadecimal System (assembly)
  - Base 16 (0..9, A, B, C, D, E, F)
    \[1234 = 1 \times 16^3 + 2 \times 16^2 + 3 \times 16^1 + 4 \times 16^0 = 4096 + 512 + 48 + 4 = 4660\]

- Hexadecimal <-> Binary
  - Four bits hold an hexadecimal digit
    \[0x0 = 0000, \ 0x1 = 0001, \ \ldots, \ 0xff = 1111\]
Numbering Systems

- Two's Complement Binary System
  - Base 2
  - Signed integers
    - Most significant bit holds the sign
      - 0 => positive, number stored as ordinary binary
      - 1 => negative, number stored as two's complement
  - Two's complement operation
    - Invert all bits (NOT)
    - Add 1
  - 4-bit
    - 0000 = 0, 0001 = 1, 1000 = -8, 1111 = -1
Numbering Systems

- Two's Complement Binary System
  - Computers can implement subtraction using the adder circuitry
    \[
    5 - 3 = 5 + -3 = 0101 + 1101 = 0010 = 2
    \]
  - Sign extension
    - n-bit number to m-bit number, with \( m > n \)
      - Copy the sign bit into the extra bits
        - Semantics is always preserved
    - \(-3\)
      - 4-bit \(\Rightarrow 1101\) (0xd)
      - 8-bit \(\Rightarrow 11111101\) (0xffd)
      - 16-bit \(\Rightarrow 111111111111101\) (0xffffd)
Bits and Bytes

- **Bit** *(Binary digit)*
  - Smallest unit of information in computing
  - A bit has a single binary value, either 0 or 1
  - Bits group to form nibbles (4 bits), bytes (8 bits) and words ($2^{(n \geq 3)}$ bits)

- **Byte**
  - Eight-bit value (today, was not always like that)
  - Bit 0 is the low-order bit or least significant bit
  - Bit 7 is the high-order bit or most significant bit
  - Can hold an ASCII character
Bytes and Words

- **Word**
  - Natural unit of data in a computer
  - Fixed-size group of bits that are handled together by the machine
  - Influence the size of registers, ULA, memory and I/O buses
  - Typical sizes: 8, 16, 32, 64 bits

```
  15 14 13 12 11 10  9  8  7  6  5  4  3  2  1  0

  high-order byte  low-order byte
```
Endianness

- **Big-endian (Motorola, SPARC, PowerPC)**
  - *Most significant byte* (MSB) is stored at the memory location with the lowest address
  
  \[
  0x12345678 \text{ at } 100
  \]

- **Little-endian (Intel)**
  - *Least significant byte* (LSB) is stored at the memory location with the lowest address
  
  \[
  0x12345678 \text{ at } 100
  \]
A Boolean algebra is a set $A$, supplied with two binary operations $\wedge$ (AND), $\lor$ (OR), a unary operation $\neg$ (NOT) and two elements $0$ (zero) and $1$ (one), such that, for all elements $a$, $b$ and $c$ of set $A$, the following axioms hold:
Boolean Algebra: Axioms

**Associativity**
\[ a \lor (b \lor c) = (a \lor b) \lor c \quad a \land (b \land c) = (a \land b) \land c \]

**Commutativity**
\[ a \lor b = b \lor a \quad a \land b = b \land a \]

**Absorption**
\[ a \lor (a \land b) = a \quad a \land (a \lor b) = a \]

**Distributivity**
\[ a \lor (b \land c) = (a \lor b) \land (a \lor c) \quad a \land (b \lor c) = (a \land b) \lor (a \land c) \]

**Complements**
Boolean Algebra: Identities

Idempotency
\[ a \lor a = a \quad a \land a = a \]

Boundedness
\[ a \lor 0 = a \quad a \land 1 = a \]
\[ a \lor 1 = 1 \quad a \land 0 = 0 \]

Complements
\[ \neg 0 = 1 \quad \neg 1 = 0 \]

De Morgan's laws
\[ \neg (a \lor b) = \neg a \land \neg b \quad \neg (a \land b) = \neg a \lor \neg b \]

Involution
\[ \neg \neg a = a \]
Truth Tables

- Can represent boolean functions
  - OR
    
    \[
    \begin{array}{c|c|c}
    OR & 0 & 1 \\
    \hline
    0 & 0 & 1 \\
    1 & 1 & 1 \\
    \end{array}
    \]
  
  - AND
    
    \[
    \begin{array}{c|c|c}
    AND & 0 & 1 \\
    \hline
    0 & 0 & 0 \\
    1 & 0 & 1 \\
    \end{array}
    \]

- Four-variable function
  
  \[F = AB + CD\]
  
  \[
  \begin{array}{c|c|c|c|c}
  BA & 00 & 01 & 10 & 11 \\
  \hline
  00 & 0 & 0 & 0 & 1 \\
  01 & 0 & 0 & 0 & 1 \\
  10 & 0 & 0 & 0 & 1 \\
  11 & 1 & 1 & 1 & 1 \\
  \end{array}
  \]
Truth Tables

- Alternative representation

<table>
<thead>
<tr>
<th>F = AB + C</th>
<th>00</th>
<th>01</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0</td>
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<td>BA</td>
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</tbody>
</table>
Truth Tables

Alternative representation

<table>
<thead>
<tr>
<th>C</th>
<th>B</th>
<th>A</th>
<th>F = ABC</th>
<th>F = AB + C</th>
<th>F = A + BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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Canonical Forms

- Allows for function simplification
  - For any given boolean function there exists a unique canonical form
- Disjunction of minterms (sums of products)
  - Minterm is a logical expression of $n$ variables consisting of only the logical conjunction operator and the complement operator
    
    $F = AB + C = ABC + A'BC + AB'C + A'B'C + ABC'$
Canonical Forms

- Conjunction of maxterms \textit{(product of sums)}
  - Maxterm is a logical expression of \textit{n} variables consisting of only the logical \textit{disjunction} operator and the \textit{complement} operator
  - Three variables: \(A'+B'+C'\), \(A+B'+C'\), \(A'+B+C'\), \(A+B+C'\), \(A'+B'+C\), \(A+B'+C\), \(A'+B+C\), \(A+B+C\)

\[ F = AB + C = (A+B+C) (A'+B+C) (A+B'+C) \]
Basic Computer Organization
Central Processing Unit

- Processor
  - Interprets instructions and operates on data

- Elements
  - Control Unit
    - Controls the CPU, fetches instructions and data for the ALU
  - Arithmetic and Logic Unit (ALU)
    - Perform arithmetic and logic operations on data stored in registers
  - Registers
    - Holds data being manipulated by the ALU
Main Memory

- Stores data and instructions
  - Von Neumann: single memory
  - Harvard: separate memories
- Read/write access
- Usually byte-addressed (not word-addressed anymore)
- Technologies
  - RAM, ROM, EEPROM, Flash
I/O Devices

- Connect the computer to the “world”
- Devices are usually connected through secondary buses (i.e. ISA, PCI, VME, USB)
- Input
  - Keyboard, mouse
- Output
  - Monitor, printer
- Storage
  - Disk, DVD, flash
- Network
  - Ethernet, Wi-Fi
System Bus

- Interconnect system components
  - CPU
  - Memory
  - I/O devices

- Signals
  - Addresses
  - Data
  - Control
    - R/W, I/O, INT, REQ, GRANT, CLOCK
Instruction Set Architecture

“An instruction set, or instruction set architecture (ISA), is the part of the computer architecture related to programming, including the native data types, instructions, registers, addressing modes, memory architecture, interrupt and exception handling, and external I/O. An ISA includes a specification of the set of opcodes (machine language), the native commands implemented by a particular CPU design.”

(Wikipedia)
Instruction Sets X CPU Storage

- Internal storage in the CPU
  - Stack (operands on the top of the stack)
  - Accumulator (single accumulator “register”)
  - Set of registers

C = A + B

<table>
<thead>
<tr>
<th>Stack</th>
<th>Accumulator</th>
<th>Register</th>
</tr>
</thead>
<tbody>
<tr>
<td>push A</td>
<td>load A</td>
<td>load r1, A</td>
</tr>
<tr>
<td>push B</td>
<td>add B</td>
<td>add r1, B</td>
</tr>
<tr>
<td>add</td>
<td>store C</td>
<td>store C, r1</td>
</tr>
<tr>
<td>pop C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Register Machines

- **Number of operands for ALU instructions**
  
  \[
  \begin{align*}
  \text{add } r1, r2 & \quad (r1 \leftarrow r1 + r2) \\
  \text{add } r1, r2, r3 & \quad (r1 \leftarrow r2 + r3)
  \end{align*}
  \]

- **Number of memory operands for ALU instructions**
  
  - **Zero (register-register)**
    
    \[
    \text{add } r1, r2 \quad (r1 \leftarrow r1 + r2)
    \]

  - **One (register-memory)**
    
    \[
    \text{add } r1, B \quad (r1 \leftarrow r1 + B)
    \]

  - **Many (memory-memory)**
    
    \[
    \text{add } C, A, B \quad (C \leftarrow A + B)
    \]